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PERIODIC REVIEW INVENTORY MODEL IN FUZZY AND INTUITIONISTIC FUZZY ENVIRONMENTS

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ABSTRACT:

In this paper, a constrained probabilistic review periodic inventory model with variable demand with constant lead time, constant demand with variable lead time and variable demand with variable lead time will be investigated. The objective is to determine the reorder point and the target inventory of the order quantity, in the light of system and cost parameters, so that the sum of all costs associated with the system will be minimized. The optimal solutions of the average demand during review period, Safety stock and target inventory which minimize the expected total cost are found using TrFN & TrIFN.

Introduction:

The influence of deterioration is so important in many inventory systems that it cannot be ignored. When we refer to an object as having deteriorated, degraded, or spoilt, we mean that it can no longer be used for its intended function; in other words, it has changed during storage to the point that it gradually loses some or all of its value.

In certain systems, the object might become out-dated due to design modifications or technological breakthroughs, but in other models, it might be lost over time and its value would decline with age. When assessing the system in any of these situations, it is imperative to consider the inventory loss resulting from deterioration since it offers a partial picture of how inventory systems function.

Certain commodities, like photographic videos, medications and medical supplies, other chemicals, and electronic components, may experience severe deterioration during the course of their regular inventory storage period. As such, the failure needs to be considered when determining their storage strategies.

For products that deteriorate with time, many probabilistic models have been developed. For Gumbel degrading items, for instance, [13] produced a periodic review inventory model when demand followed a pareto distribution. A probabilistic inventory model featuring a pareto demand distribution and a two-parameter exponential degrading rate was examined by [12]. [14] presented an inventory model with time-dependent demand and shortages that included Weibull deterioration for decaying items.

A partial backlog and quadratic demand inventory model for decaying items was presented by [4]. [3] investigated the best way to manage an exponentially degrading production inventory model. [6] made with a probabilistic scheduling period inventory system for items with a lead time of one scheduling period that are destined to decay over time.

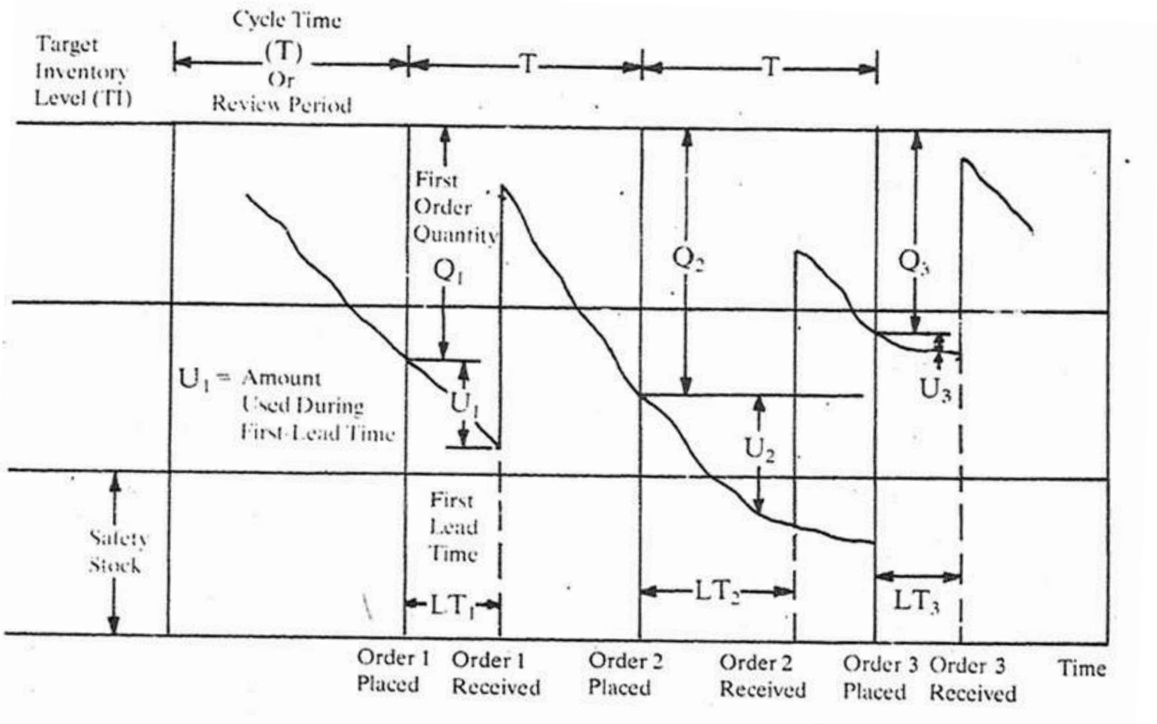
[5] investigated the scheduling-period inventory model for in-demand and quickly degrading goods. For a system with a constant rate of deterioration, a comment on an order-level inventory model was explained by [2]. Numerous probabilistic inventory models presuppose that either one of the cost units is different or that the cost units remain constant. Models for this are presented in hundreds of books and articles under various assumptions and settings.

[10] investigated a probabilistic multi-item inventory model with variable mixture shortage cost under restrictions. [9] used the Lagrange approach and fuzzy adaptive particle swarm optimization to construct a probabilistic periodic review inventory model.

[11] looked at a probabilistic multi-item inventory system that was subject to limitations and fluctuating ordering costs, but had frequent reviews and zero lead time. Introduce the two linear constraints on a probabilistic single-item inventory problem with variable order cost [8]. Under two

constraints, [1] examined a probabilistic multi-item inventory model with variable order costs. [7] provided an explanation of the theory and analysis of the inventory and procurement system.

Periodic Review system Model development:
The amount required to bring the ordered and available stock up to a desired level is known as the replenishment quantity, which is a variable. Under this approach, replenishment orders are placed at the conclusion of each evaluation period, and the stock levels are checked at a preset interval of time known as the review period. Hence the key variable for system are the review period $\tilde{T}$ and the largest inventory level $\tilde{T}_L$. Other names for this system include the P-system, the Replenishment Inventory System, and the Fixed Period System.



Order Quantity=Target-Inventory-Inventory on Hand

The way a forced ordering review system works is that, as seen in the above image, an order must be placed at the end of every review session. Every review session lasts precisely T days, after which an order is placed for an amount big enough to bring inventories back up to the goal level $\tilde{T}$. The cargo arrives and is placed into inventory after the reorder lead time (t_L). Every period, the order quantities are determined by subtracting the inventory on hand from the prior orders that have not yet been filled, using the formula $Q = \tilde{T}$. The following is the target inventory formula for a set interval T.

$$Target\ Inventory\ = \begin{pmatrix} Average\ review \\ period\ demand \end{pmatrix} + \begin{pmatrix} Average\ lead \\ time\ demand \end{pmatrix} + Buffer\ stock$$

(i) Levels of Control for Constant Lead Time and Variable Demand:

Over the lead and review periods, the average demand is $\tilde{d}(\tilde{T} + \tilde{t_L})$, where $\tilde{t_L}$ is the reorder – lead time. During this period, the Buffer stock is

$$B = Z\sigma_{\tilde{d}}\sqrt{(\tilde{T} + \tilde{t_L})}.$$

where Z is the quantity of standard deviations required to provide the required level of service.
The target Inventory level is

$$\tilde{T_L} = \tilde{d}(\tilde{T} + \tilde{t_L}) + Z\sigma_{\tilde{d}}\sqrt{(\tilde{T} + \tilde{t_L})}$$

(ii) Levels of Control for Variable Lead Time and Constant Demand:

Mean demand during the duration of the review $= \tilde{T}\tilde{d}$
Mean demand throughout the lead time $= \tilde{d} * \overline{\tilde{t_L}}$

Safety Stock = B = $Z\tilde{d}\sigma_{\tilde{t}_L}$

Thus,

$$\tilde{T_L} = \tilde{T}\tilde{d} + \tilde{d} * \tilde{t}_L + Z\tilde{d}\sigma\tilde{t}_L$$
$$= \tilde{d}(\tilde{T} + \tilde{t}_L) + Z\tilde{d}\sigma\tilde{t}_L$$

Where, $\tilde{t}_L$ = Standard deviation of lead time.

(iii) Levels of Control for Variable Lead Time and Variable Demand:

Mean demand during the duration of the review $= \tilde{T}\tilde{d}$
Mean demand throughout the lead time $= \tilde{d} * \tilde{t}_L$

$$Buffer(or)\ stock = Z\sqrt{\tilde{T}\ var(\tilde{d}) + Var(\tilde{U})}$$
$$var(\tilde{d}) = \sigma_{\tilde{d}}^2 = Variance\ of\ daily\ demand$$
$$var(\tilde{U}) = \sigma_{\tilde{U}}^2 = Variance\ of\ lead\ time\ demand$$
$$= Var(\tilde{d})\tilde{t}_L + Var(\tilde{t}_L)\ \tilde{d}^2$$
$$\tilde{T_L} = \tilde{d}(\tilde{T} + \tilde{t}_L) + Z\sqrt{\tilde{d}^2\sigma\tilde{t}_L^2 + \sigma_{\tilde{d}}^2(\tilde{T} + \tilde{t}_L)}$$

Generally, the review period $\tilde{T} = \sqrt{\frac{2\widetilde{C_o}}{\tilde{D}\widetilde{C_h}}}$

4.1.1. Numerical Example:

An item's daily demand follows a normal distribution with a mean of 50 units and a standard deviation of 5 units. There's a lead time of six days. Unit prices range from Rs 1.00 to Rs 1.20, while order placement fees range from Rs5 to Rs 8. Annual holding cost range from Rs 0.24 to Rs. 0.28. 95% of services is the desired level. There is no stock-out cost, but back orders are accepted. Determine

(i) the various inventory levels.

(ii) the target inventory level for the renew period “T”.

Solution:

$$Buffer(or)\ stock = Z\sqrt{\tilde{T}\ var(\tilde{d}) + Var(\tilde{U})}$$
$$var(\tilde{d}) = \sigma_{\tilde{d}}^2 = Variance\ of\ daily\ demand$$
$$var(\tilde{U}) = \sigma_{\tilde{U}}^2 = Variance\ of\ lead\ time\ demand$$
$$= Var(\tilde{d})\tilde{t}_L + Var(\tilde{t}_L)\ \tilde{d}^2$$
$$\tilde{T_L} = \tilde{d}(\tilde{T} + \tilde{t}_L) + Z\sqrt{\tilde{d}^2\sigma\tilde{t}_L^2 + \sigma_{\tilde{d}}^2(\tilde{T} + \tilde{t}_L)}$$

Generally the review period $\tilde{T} = \sqrt{\frac{2\widetilde{C_o}}{\tilde{D}\widetilde{C_h}}}$

Costs as TrFN:

Normally, the review period T is set as

$$\tilde{T} = \sqrt{\frac{2\widetilde{C_o}}{\tilde{D}\widetilde{C_h}}}$$
$$\widetilde{C_o} = (5,6,7,8)$$
$$\widetilde{C_h} = (0.24,0.25,0.26,0.28)$$

$$\frac{2\widetilde{C_o}}{\tilde{D}\widetilde{C_h}} = \frac{2 * (5,6,7,8)}{50 * 365 * (0.24,0.25,0.26,0.28)}$$
$$= \frac{(10,12,14,16)}{(4380,4562.5,4745,5110)}$$

$$= \left(\frac{10}{5110}, \frac{12}{4745}, \frac{14}{4562.5}, \frac{16}{5110}\right)$$
$$= (0.0020, 0.0025, 0.0031, 0.0031)$$
$$\tilde{R}(0.0020, 0.0025, 0.0031, 0.0031) = 0.0027$$
$$\tilde{T} = \sqrt{0.0027} = 0.052 = 18.98 = 19\ days$$

Six days are the lead time.

Mean demand during the duration of the review $\tilde{T}$ = 19 X 50 = 950 units

Mean demand throughout the lead time, $\tilde{t}_L$ = 6 X 50 = 300 units

Safety stock $B = Z\sigma_{\tilde{d}}\sqrt{(\tilde{T} + \tilde{t_L})}$= 1.645 X 5 X 4.899 = 40.29 = 40 units

Thus, the target inventory level

$\widetilde{T_L}$ = 950+300+40 = 1290 Units.

Costs as TrIFN:

Normally, the review period T is set as

$$\tilde{T} = \sqrt{\frac{2\widetilde{C_o}}{\tilde{D}\widetilde{C_h}}}$$
$$\widetilde{C_o} = (5, 5.25, 6, 7, 7.75, 8)$$
$$\widetilde{C_h} = (0.24, 0.245, 0.25, 0.26, 0.275, 0.28)$$
$$\frac{2\widetilde{C_o}}{\tilde{D}\widetilde{C_h}} = \frac{2 * (5, 5.25, 6, 7, 7.75, 8)}{50 * 365 * (0.24, 0.245, 0.25, 0.26, 0.275, 0.28)}$$
$$= \frac{(10, 10.5, 12, 14, 15.5, 16)}{(4380, 4471.25, 4562.5, 4745, 5018.75, 5110)}$$
$$= \left(\frac{10}{5110}, \frac{10.5}{5018.75}, \frac{12}{4745}, \frac{14}{4562.5}, \frac{15.5}{4471.25}, \frac{16}{5110}\right)$$
$$= (0.0020, 0.0021, 0.0025, 0.0031, 0.0035, 0.0031)$$
$$\tilde{R}(0.0020, 0.0021, 0.0025, 0.0031, 0.0035, 0.0031) = 0.0028$$
$$\tilde{T} = \sqrt{0.0027} = 0.053 = 19.345 = 19\ days$$

Lead time = 6 days

Mean demand during the duration of the review $\tilde{T}$ = 19 X 50 = 950 units

Mean demand throughout the lead time, $\tilde{t}_L$ = 6 X 50 = 300 units

Safety stock = $B = Z\sigma_{\tilde{d}}\sqrt{(\tilde{T} + \tilde{t_L})}$= 1.645 X 5 X 4.899 = 40.29 = 40 units

Thus, the target inventory level

$\widetilde{T_L}$= 950+300+40 = 1290 Units.

4.1.2. Numerical Example:

A business has set a one-stock-out every three years service level for an item and the review period is fixed for 90 days. The following table displays the upper and lower bounds of the reorder lead times. There is a 45-unit daily demand for the item. Determine the target inventory level.

Month	January	February	March	April	May	June
Limits of lead – time	(6, 9)	(11, 14)	(23, 26)	(15, 18)	(12, 15)	(14, 17)

Solution:

Take the lead time as TrFN and defuzzify the fuzzy numbers:

Month	January	February	March	April	May	June
Limits of lead – time	(6,7,8,9)	(11,12,13,14)	(23,24,25,26)	(15,16,17,18)	(12,13,14,15)	(14,15,16,17)
Average lead – time	7.5	12.5	24.5	16.5	13.5	15.5

Average lead time $\widetilde{t_L} = \frac{7.5+12.5+24.5+16.5+13.5+15.5}{6} = 15$ days.

Average demand during average lead time = 45 X 15 = 675 units

Variance of lead time = $\frac{(7.5-15)^2+(12.5-15)^2+(24.5-15)^2+(16.5-15)^2+(13.5-15)^2+(15.5-15)^2}{6}$

$$= \frac{157.5}{6} = 26.25\ days$$

Average demand during review period = 90 X 45 = 4050 units.

Safety stock for the service level of 91.7% confidence = 1.39 X 45 X $\sqrt{26.25}$ = 320.56

The target inventory level is

$\widetilde{T_L}$= 4050 + 675 + 320.56 = 5045.56 = 5046 units.

Take the lead time as TrIFN and defuzzify the fuzzy numbers:

Month	January	February	March	April	May	June
Limits of lead-time	(6, 6.2, 7, 8, 8.7, 9)	(11,11.2,12, 13,13.7, 14)	(23,23.2,24, 25,25.7,26)	(15,15.2,16,17,17.7, 18)	(12,12.2,13, 14,14.7,15)	(14,14.2,15, 16,16.7, 17)
Average lead- time	7.48	12.48	24.48	16.48	13.48	15.48

Average lead time $\widetilde{t_L} = \frac{7.48+12.48+24.48+16.48+13.48+15.48}{6} = 14.98$ days.

Average demand during average lead time = 45 X 14.98 = 674.1 units

Variance of lead time

$= \frac{(7.48-14.98\ )^2+(12.48-14.98\ )^2+(24.48-14.98\ )^2+(16.48-14.98\ )^2+(13.48-14.98\ )^2+(15.48-14.98\ )^2}{6}$

$= 26.25\ days$

Average demand during review period = 90 X 45 = 4050 units.

Safety stock at 91.7% confidence level for the service level = 1.39 X 45 X $\sqrt{26.25}$ = 320.56

The target inventory level is

$\widetilde{T_L}$ = 4050 + 674.1 + 320.56 = 5044.66 = 5045 units.

4.1.3 Numerical Example:

Consider the example 4.1.1 & 4.1.2 and determine the Target inventory level.

Solution:

Take the lead time as TrFN and defuzzify the fuzzy numbers:

Average lead time $\widetilde{t_L} = 15$ days.

Variance of lead time $= 26.25\ days$

Review period T = 90 days

Average consumption during review period = 90 x 45 = 4050 units

Average consumption during average lead time = 45 x 15 = 675

Safety stock = $Z\sqrt{\tilde{d}^2\sigma_{\tilde{t}_L}^2 + \sigma_{\tilde{d}}^2(\tilde{T} + \tilde{t}_L)}$= 402.28 units

$$Target\ Inventory\ Level = \begin{pmatrix} Average\ review \\ period\ demand \end{pmatrix} + \begin{pmatrix} Average\ lead \\ time\ demand \end{pmatrix} + Buffer\ stock$$

Target inventory level = 4050 +675 + 402.28 = 5127.28 units.

Take the lead time as TrIFN and defuzzify the fuzzy numbers:

Average lead time $\widetilde{t_L} = 14.98$ days.

Variance of lead time $= 26.25\ days$

Review period T = 90 days

Average consumption during review period = 90 x 45 = 4050 units

Average consumption during average lead time = 45 x 14.98 = 674.1

Safety stock = $Z\sqrt{\tilde{d}^2\sigma_{\tilde{t}_L}^2 + \sigma_{\tilde{d}}^2(\tilde{T} + \tilde{t}_L)}$= 402.23 units

$$Target\ Inventory\ Level = \begin{pmatrix} Average\ review \\ period\ demand \end{pmatrix} + \begin{pmatrix} Average\ lead \\ time\ demand \end{pmatrix} + Buffer\ stock$$

Target inventory level = 4050 +674.1 + 402.23 = 5126.33 units.

Conclusion:

In this paper, probabilistic review periodic inventory model is considered in fuzzy and intuitionistic fuzzy environments. Discussion is held on three scenarios: variable demand with constant lead time, constant demand with variable lead time, and variable demand with variable lead time. The proposed approach eliminates the impreciseness uncertainty prevailing in periodic review inventory models.

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